A two-stub filter derived from an exact maximally flat design was constructed in strip-line [5]. The stop-band width is five per cent of the stop-band center frequency $f_0=1.6$ Gc/s, and the basic filter has a maximally-flat response. The element values of the low-pass prototype are $g_0=g_3=1$, and $g_1=g_2=1.414$. The exact strip-line filter has stub impedances $Z_1=950$ ohms, and $Z_2=900$ ohms, and a connecting-line impedance $Z_{12}=53$ ohms, and 50-ohm terminations. The modified filter is shown in Fig. 4(a), which also shows its computed and measured attenuation loss. Figure 4(b) shows the measured and computed VSWR for a small region near the first stop band. A sketch of the filter is given in Fig. 5, and a photograph is shown in Fig. 6, the cover plate having been removed in both cases. Tuning screws and stub supports with provision for adjusting the length of each resonator permit the coupling gaps and the resonant lengths to be independently adjusted. In this two-resonator design, the second stop band is seen to be considerably wider than the first stop band, unlike the exact design on which it is based. Nevertheless, this design method appears to be useful and capable of yielding accurate designs of practical narrow-band microwave band-stop filters.

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Coupled Rods Between Ground Planes

In a recent paper, Cristal [1] provided graphs of self- and mutual-capacitances (C_g/ϵ and C_M/ϵ) of parallel round conductors between two ground planes as a function of rod separation distance s/b, with rod diameter d/b as a parameter. To realize a distributed element filter, the self- and mutual-capacitance values are first calculated [2], [3]. Then the separation distances and rod diameters are found by using Cristal's graphs which are based on the assumption that the charge on the rod is equally distributed between facing ground planes and adjacent rods.

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The subject of this correspondence is to show that after calculating the capacitance values as above, Honey's approximations [4] can be used to determine analytically the geometry of coupled-rod structures with favorable results.

Honey's approximations are:

$$Z_{oe}-Z_{oo}=\frac{120}{\sqrt{\epsilon r}}\ln\coth\frac{\pi s}{2b}$$

$$Z_{oe}+Z_{oo}=\frac{120}{\sqrt{\epsilon r}}\ln\coth\frac{\pi d}{4b}$$

The first filter using Honey's approximation was designed and built in 1962 by E. Cota of these laboratories. Since then various filters in round bar configuration have been realized, ranging from 0.3 per cent bandwidth comb line structure to 72 per cent interdigital types.

The transformation of Matthaei's 10 per cent interdigital filter into circular rods between ground planes by using Honey's approximations is carried out in the Appendix. The transformation calculations, together with Matthaei's filter synthesis procedures, are easily programmed on a computer, thereby eliminating tedious manual calculations.

A comparison of Cristal's procedure and this analytic method is made in Table I.

From the table it is readily seen that the separation distances $s_{k,k+1}/b$ agree better than the rod diameters d_k/b. This method yields slightly higher even and odd mode impedances than Cristal's procedure. Since in this type of geometry, the optimum Q corresponds to a broad range of impedance values, the filter performances using either one of the above realizations are essentially equivalent. Obviously, a round bar configuration will yield a higher Q and is easier to fabricate as pointed out by Cristal.

The end resonators by this method are found to be bigger (comparative to Getsinger's bars) and their respective separation distance to the adjacent resonator smaller than Cristal's procedure. However, Cristal found that the separation distance between the end resonators required empirical adjustment to a value of 0.625 inch closely agreeing with the 0.623 inch value computed with the aid of Honey's approximation.

Thus the described method can be manually calculated or programmed on a computer to yield directly rod diameter and separation distance values that require no graphical interpolation or empirical adjustment.

APPENDIX

The following equations are given:

$$Z_{oe}^k-Z_{oo}^k=\frac{120}{\sqrt{\epsilon r}}\ln\coth\frac{\pi}{2}\frac{s_{k,k+1}}{b} \qquad (1)$$

$$Z_{oe}^k+Z_{oo}^k=\frac{120}{\sqrt{\epsilon r}}\ln\coth\frac{\pi}{4}\frac{d_k}{b} \qquad (2)$$

$$Z_{oo}^k=\frac{C_{k+1}}{vF_{k,k+1}} \qquad (3)$$

$$Z_{oe}^k=\frac{C_{k+1}+2C_{k,k+1}}{vF_{k,k+1}}=Z_{oo}^k+\frac{2C_{k,k+1}}{vF_{k,k+1}} \qquad (4)$$

where,

$$F_{k,k+1}=C_kC_{k+1}+C_{k,k+1}(C_k+C_{k+1}) \qquad (5)$$

v = velocity of light in medium of propagation.

From (1) to (5) we get:

$$Z_{oe}^k-Z_{oo}^k=\frac{2C_{k,k+1}}{vF_{k,k+1}}=\frac{120}{\sqrt{\epsilon r}}\ln\coth\frac{\pi}{2}\frac{s_{k,k+1}}{b} \qquad (6)$$

$$\frac{60}{\sqrt{\epsilon r}}\ln\coth\frac{\pi}{2}\frac{s_{k,k+1}}{b}=\frac{C_{k,k+1}}{vF_{k,k+1}} \qquad (6')$$

$$Z_{oe}^k+Z_{oo}^k=2\left(\frac{C_{k+1}+C_{k,k+1}}{vF_{k,k+1}}\right) \qquad (7)$$

$$\frac{60}{\sqrt{\epsilon r}}\ln\coth\frac{\pi}{4}\frac{d_k}{b}=\overline{Z_{oo}^k}+\overline{\frac{C_{k,k+1}}{vF_{k,k+1}}} \qquad (7')$$

The impedance-capacitance relationships (3) and (4) define $N-1$ equalities for a set of N elements; thus the separation distances $s_{k,k+1}/b$ are uniquely determined from (6'). In the case of rod diameters, we resort to an averaging process to get N equalities from $N-1$ relationships (7'). The average impedance of a circular rod in this case is computed from the average of the self and mutual capacitances looking to the right and left from the respective circular rod. Thus we rewrite (7') into the following form:

$$\frac{60}{\sqrt{\epsilon r}}\ln\coth\frac{\pi}{4}\frac{d_k}{b}=Z_{oo}^k+\frac{C_{k,k+1}}{vF_{k,k+1}} \qquad (8)$$

which yields N equations for N elements, as required.

In the case of the end resonators, we include the fringing capacitance to one side. From Fig. 1, using (3), we can write:

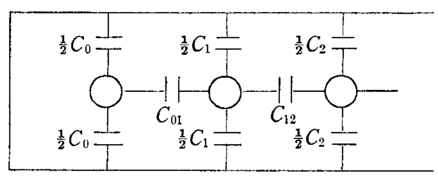

Fig. 1.

Z_{oo}^0 left $=0$, provided the filter housing end plate is sufficiently far removed,

$$Z_{oo}^0\text{ right}=\frac{C_1}{vF_{01}}=\frac{1}{v}\frac{C_1}{C_0C_1+C_{01}(C_0+C_1)}$$

Thus

$$\overline{Z_{oo}^0}=1/2(Z_{oo}^0\text{left}+Z_{oo}^0\text{right})=1/2(Z_{oo}^0) \qquad (9)$$

The remaining $\overline{Z_{oo}^k}$'s, that is from Z_{oo}^1 to Z_{oo}^{k-1} are

$$\overline{Z_{oo}^k}=1/2(Z_{oo}^k\text{left}+Z_{oo}^k\text{right}) \qquad (10)$$

From Matthaei's distributed element filter synthesis, we obtain C_k/ϵ and $C_{k,k+1}/\epsilon$ instead of C_k, and $C_{k,k+1}$, thus (3) and (4) become:

TABLE I

COMPARISON OF THE TWO PROCEDURES TO ARRIVE AT ROUND BAR DIAMETER AND THEIR SEPARATION DISTANCES

k	This design d_k/b	Cristal's design d_k/b	Getsinger's rectangular bars $w/b \times t/b$	k, $k+1$	This design $s_{k,k+1}/b$	Cristal's design $s_{k,k+1}/b$
0&7	0.797	0 516	0.648 by 0.3	0, 1&7, 6	0.623	0.638
1&6	0.278	0 324	0.243 by 0.3	1, 2&6, 5	1.009	1.003
2&5	0.339	0.352	0.294 by 0.3	2, 3& 5, 4	1.087	1.114
3&4	0.344	0.351	0.293 by 0.3	3, 4	1.103	1.125

TABLE 2

CAPACITANCE'S FROM MATTHAEI'S SYNTHESIS PROCEDURE AND THE RESULTING AVERAGED IMPEDANCE VALUES

Capacitances					Impedances			
k	$\frac{C_k}{\epsilon}$	k, $k+1$	$\frac{C_{k,k+1}}{\epsilon}$	$\frac{F_{k,k+1}}{\epsilon^2}$	k	$\frac{Z_{oo}{}^k}{\text{Zint.}} \times 10^{-2}$	$\frac{C_{k,k+1}}{F_{k,k+1}} \times \frac{10^{-2}}{\text{Zint.}}$	$\frac{Z_{oe}{}^k}{\text{Zint.}} \times 10^{-2}$
0&7	5.950	0, 1&6, 7	1.582	34.966	0&7	4.850	4.526	13.902
1&6	3.390	1, 2&5, 6	0.301	17.335	1&6	21.261	3.181	27.623
2&5	4.420	2, 3&4, 5	0.226	21.887	2&5	20.048	1.384	22.816
3&4	4.496	3, 4	0.218	22.174	3&4	20.234	1.007	22.248

TABLE 3

ARGUMENTS LEADING TO ROD DIAMETERS AND SEPARATION DISTANCES (CENTER TO CENTER)

Rod Diameter			Separation Distances		
k	$\frac{10^{-2}}{\text{Zint.}}\left[Z_{oo}{}^k + \frac{C_{k,k+1}}{F_{k,k+1}}\right]$	$\frac{d_k}{b}$	k, $k+1$	$\frac{10^{-2}}{\text{Zint.}}\left[\frac{C_{k,k+1}}{F_{k,k+1}}\right]$	$\frac{s_{k,k+1}}{b}$
0&7	9.376	0.797	0, 1&6, 7	4.526	0.623
1&6	24.442	0.278	1, 2&5, 6	1.736	1.009
2&5	21.432	0.339	2, 3&4, 5	1.032	1.087
3&4	21.241	0.344	3, 4	0.983	1.103

$$Z_{oo}{}^k = \text{Zint.}\ \frac{C_{k+1/\epsilon}}{F_{k,k+1/\epsilon}{}^2} \qquad (3')$$

$$Z_{oe}{}^k = \text{Zint.}\ \frac{C_{k+1/\epsilon} + 2C_{k,k+1/\epsilon}}{F_{k,k+1/\epsilon}{}^2} \qquad (4')$$

where Zint. = intrinsic impedance of free space.

The calculations, leading to the realization of Matthaei's 10 per cent bandwidth interdigital filter [4] on page 484 in round bar configuration are summarized in Tables II and III.

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Author's Comment

The comparison of interdigital filter designs of Vadopalas [1] and Cristal [2] given in Table I of Vadopalas shows several differences as well as similarities. Although the center-to-center spacings of the rods of the two designs are in close agreement, a comparison of the spacings between rod surfaces gives the following (rounded) percentage values.

$S_{k,k+1}/b$	Percentage difference in designs
S_{01}/b and S_{67}/b	60
S_{12}/b and S_{56}/b	5
S_{23}/b and S_{45}/b	2
S_{34}	2

The comparison of the rod diameters of the two designs shows the following results.

d_k/b	Percentage difference in designs
d_0/b and d_7/b	54
d_1/b and d_6/b	16
d_2/b and d_5/b	4
d_3/b and d_4/b	2

The large deviations between the spacings of Rods (0, 1) and (6, 7) and the diameters of Rods (0, 7) and (1, 6) can be attributed partially to the inaccuracy of the formulas of Honey [3] upon which the method of Vadopalas rests. The formulas of Honey are believed to be accurate [4] for values of $d/b < 0.25$ and c/b (normalized center-to-center spacing) greater than $3(d/b)$. Examination of the normalized design parameters of Rods 1, 2, and 3 given in Table I [1] shows them to be near the stated bounds required by the formulas of Honey. On the other hand, the normalized parameters of Rods (0, 1) and spacings to adjacent rods exceeds the bounds of the variables in Honey's formulas. It is therefore not unreasonable that the two interdigital filter designs are somewhat alike where the range of design parameters are small enough to justify using Honey's formulas, and that the two designs are considerably different when the formulas of Honey are invalid.

Still, aside from the approximate nature of Honey's formulas (as well as the fact that they are intended only for pairs of rods and not arrays of rods) it is curious that the two designs agree to the extent that they do. The design equations in Vadopalas state that the center-to-center spacings of the rods are independent of the rod diameters (6'); and that the rod diameters are independent of the center-to-center spacings of the rods (8, 9, and 10). This is in contradistinction to the design procedure in [2], which takes into account the interdependence of these quantities.

The question of which design procedure to use is perhaps best resolved by the performance of the resulting filters. The design procedure in Cristal corresponds closely to the physics and the geometry of the filter. The excellent correspondence between the responses of the constructed filter (Fig. 16, Cristal [2]) and the theoretically expected values would seem to bear this out. The measured response of other filters based on this design procedure have also been extremely close to their theoretically expected values [5].

In conclusion, it may be stated that for workers who require a large number of designs of interdigital or comb-line filters, the method of Vadopalas when programmed on a digital computer has definite computational advantages. Users of this method, however, should be aware of the limitations of the formulas of Honey. They were not intended for the design of filters using arrays of coupled rods, and they are inaccurate for normalized rod diameters greater than 0.25, or for closely spaced rods.

On the other hand, the design method given in [2], although graphical, corresponds closely to the related physics and has resulted in filters that yield responses extremely close to theoretical. The method may also be applied to the design of structures other than interdigital and comb-line filters such as parallel-coupled-resonator filters, spur-line filters, and coupled-TEM-mode transmission line directional couplers for which the formulas of Honey might not be applicable.

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